Performance Analysis of Three Likelihood Measures for Color Image Processing

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Abstract

Image segmentation is a low–level operation, concerned with partitioning an image into homogeneous regions. In a large number of applications, segmentation plays a fundamental role for the subsequent higher–level operations; such as recognition, object–based image/video compression, object tracking, scene analysis, and object–based image editing. Until recently, attention was focused on segmentation of grayscale images, but the advances in computational power and instrumentation has evolved the research on color image segmentation. Although, many researchers have tried to extend the methods of grayscale image segmentation to the color images, working in this multidimensional field, enables the implementation of more efficient methods. At the heart of any color image segmentation method, relies an appropriate likelihood measure or a suitable homogeneity criteria. In this paper, the performance of the three paradigms of *Euclidean distance*, *Mahalonobis distance*, and *reconstruction error* are analyzed in terms of achieving perfect likelihood measures, robustness, and leading to promising homogeneity decisions. While the Euclidean distance is proved to perform poor in all cases, the proposed reconstruction error out–performs the Mahalonobis distance with much lower computation cost.

Key words: Principle Component Analysis, Color Image Processing, Segmentation, Fuzzy Sets, Mahalonobis Distance, Euclidean Distance.

1 Introduction

Image segmentation is a low–level operation, concerned with partitioning an image into homogeneous regions [1–3]. Although, there are many different descriptions for homogeneity, such as the intensity, texture, and color, to name a few, color

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is known as a more appropriate homogeneity descriptor, because of its simplicity and intuitionism [4]. In a Large number of applications (in image processing, computer vision, and computer graphics), segmentation plays a fundamental role before applying the higher–level operations like recognition, object–based image/video compression, object tracking, scene analysis, and object–based image editing [5,6].

As image segmentation is a *spatial–spectral* process (finding clusters in the spectral histogram which result in meaningful homogeneous regions in the spatial domain), the problem is solved in a *give–and–take* fashion between satisfying (sometimes) contradictory spectral and spatial concerns. It is proved that any image segmentation approach must solve the problem of finding either a suitable likelihood measure or a meaningful region homogeneity criteria [4]. In this paper, we refer to the problem of finding a meaningful likelihood measure and a region homogeneity criteria.

Some simple methods are reported in the literature, that extend grayscale segmentation methods to the color image segmentation era (*e.g.* see [7]). Investigating this *extended–grayscale* methods makes clear that they are also using a min/max fashion likelihood measure.

Using the Euclidean distance is generally accepted as a proper likelihood measure, while different authors add some extra processing to improve the results [4]. For example in [8] the author uses the $CIE-Lab$ color space. Having in mind that the Euclidean distance only depends on the *central* point of the cluster and not on the marginal values and the spread of it, it is clear that (theoretically) The Euclidean distance is a simple but not a perfect solution. Although in [9] the author claims to use a better designed fuzzy scheme in the $CIE-Lab$ space, at the heart of the method the Euclidean distance of the point and the cluster center are ruling the likelihood. In [10] the authors uses the Euclidean distance without any manipulations.

To overcome the shortcomings of the Euclidean distance–based likelihood measures, in [11] the authors uses a weighted Euclidean distance, which when tuned well, converges to the well–known *Mahalonobis* distance. In this approach, not only the central point of the cluster is considered, but also the spread of the data is incorporated as a set of normalizing factors.

As the likelihood measures tend to compute the distance between a point and a cluster, they rank better members with smaller numbers, as is the purpose of any distance measure. This is in fact in contrast with the ordinary fuzzyfication approach which marks better members with larger numbers in the range of $[0, 1]$ and vice versa. Thus, a mapping function is needed to produce the desired result. The general definition of such a mapping is $y = f(\frac{x}{\sigma})$, while f is an odd function, mapping $[-\infty, \infty] \rightarrow [0, 1]$ in an upside down scheme. Considering a cluster r, on which the distance function is de-

fined as $e_r(\vec{c})$, leads to the likelihood measure of $f(\frac{e_r(\vec{c})}{\sigma})$. It is clear that for all $\vec{c} \in r$, we expect the likelihood measure to be near 1. Thus, σ must be statistically computed in terms of $\{e_r(\vec{c})|\vec{c} \in r\}$ while f must have a flat ceil; we expect f to map $[-1, 1]$ to the vicinity of 1. Note that, this is in contrast with the generally–used *Gaussian* functions [10].

The main purpose of this paper is to compare the three likelihood definitions of *Euclidean* distance ($\tilde{e}^E_{r,p}(\vec{c})$), *Mahalonobis* distance ($\tilde{e}^M_{r,p}(\vec{c})$), and the proposed reconstruction error [12] ($\tilde{e}^R_{r,p}(\vec{c})$), in terms of image fuzzyfication and homogeneity decision. In this paper we compare $\tilde{e}^E_{r,p}(\vec{c})$, $\tilde{e}^{\boldsymbol{M}}_{\vec{r}}$, $\tilde{e}$

processing see [17,12].

Although RGB color space is used in this paper, but having in mind that the PCA representation is translation, rotation, and scale invariant [16], while all measures and criteria investigated in this paper rely on PCA, it is clear that the conclusions of this paper applies to any color space, linearly derived from RGB in a reversible fashion [18] (such as $CMYK$ [19], YC_bC_r [20], YIQ [21], YUV [22], and $I_1I_2I_3$ [23]). This does not hold for non–linearly derived color spaces (e.g. HSI [24], HSV [25,26], $CIE-XYZ$, $CIE-La^*b^*$, $CIE-Lu^*v^*$, $CIE-L^*H^oC^*$ [27–29], and $HMMD$ [30]) or irreversible linear ones.

3 Experimental Results

To compare the performance of different likelihood measures, 42 color images (including the standard images *Lena*, *Mandrill*, *Airplane*, *Peppers*, *Girl*, and *Couple*), in RGB color space were used (all low–compressed *jpeg* files). In each image 10 rectangular homogeneous regions were selected, and the corresponding fuzzyfication results were compared subjectively. To test the robustness of the methods, tests were performed with regions containing up–to 25% irrelevant points and with different values of p. Also, the complexity of the methods were computed in terms of *flops*. To compare different homogeneity criteria, 49 randomly selected rectangular regions in each test image, were sorted according to each homogeneity criterion in a descending fashion. All experiments were performed on a large database of color images [31] and some samples representing the whole results were selected for better illustration of the results.

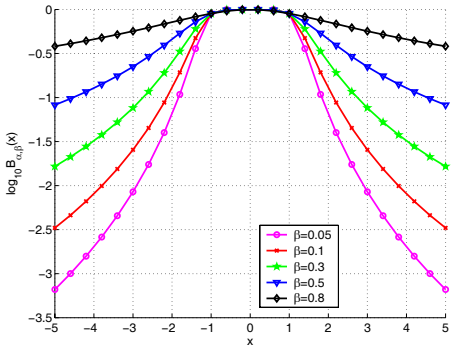


Fig. 1. Log–magnitude of the membership function ($B_{\alpha,\beta}$) for $\alpha = 0.9$ with different values of β.

Figure 1 shows the log–magnitude of the membership function, $B_{\alpha,\beta}$ for $\alpha = 0.9$ with different values of β. It is clear that while larger values of β increases the domain of the typical members (those with larger likelihood values), the function has an area of almost-one values for the interior points of the cluster. This event does not happen for the Gaussian function, in which one must increase the domain of better members, by increasing σ, leading to an overall increase of membership values. Also, the sharp transition between the points $(1, \alpha)$ and $(2, \beta)$, which enables a *crisp* classification of points in the proposed membership functions, does not exist in the Gaussian function.

The computational complexity of each likelihood method can be sep-

arated into two phases of: *data–collection* and *utilization*. It is clear that in terms of data–collection complexity, the Euclidean distance is the simplest (just needs the expectation of the cluster), while the Mahalonobis distance is the most complex (relies on computation of the expectation and also all eigenvalues and eigenvectors). The utilization phase of the Euclidean distance consists of $3 + 3 + 1 = 7$ flops, while the Mahalonobis distance needs $2 \times 9 \times (3 + 3) + 3 = 111$ flops, while the reconstruction error uses only $3+3+3+3+3+7 = 22$ flops. Thus, in terms of utilization complexity, the Mahalonobis distance is the more consuming, while the Euclidean is the fastest. The same occurs for the amount of the memory needed to *code* the likelihood measure of a region in the three methods (3, 12, and 6 real numbers for the three methods of Euclidean distance, Mahalonobis distance, and reconstruction error, respectively).

Figure 2 shows the results of fuzzyfication one of the samples with these three methods, based on four typical base regions. In all cases, the parameters are set as $\alpha = 0.95$, $\beta = 0.4$, $p = 0.9$. Note that in the entire paper, to give a better printing result, the fuzzyficated images are printed in an inverted fashion. It is clear that while non of the figures 2–b, 2–f, 2–j, and 2–n (samples of The Euclidean fuzzyfication) have a desirable fuzzyfication and do not enabling the user to classify members and non members with a reasonable margin of certainty, samples corresponding to the Mahalonobis distance and the reconstruction error are promising.

Figure 3 shows a few results of comparing the robustness of different methods against partially non–homogeneous base regions. In this test, base regions were contaminated with at most 25% irrelevant points. In figure 3–a a small portion of the leaf is added to the apple, resulting in a desperate Euclidean distance–based fuzzyfication show in figure 3–b, which both include the apple and the leaf. While the two other methods led to more acceptable results, the reconstruction error gave a better neglecting of the irrelevant points (compare Figures 3–c and 3–d). The same happens for the case of initializing all methods with almost entirely grass points and a small portion of the red ribbon (Figure 3–e). The Euclidean distance not only has failed to reject ribbon points, it also have marked the apple points with high likelihood (Figure 3–f). The two other methods have better compensated the effect of irrelevant point. Comparing Figures 3–g and 3–h makes it clear that the reconstruction error has neglected both the ribbon points and the apple points better than the Mahalonobis distance.

Figure 4 shows the results of comparing the robustness of the three different methods against parameter selection. As the effects of selecting parameters α and β is the same for all methods, here just the effect of selecting p is investigated. Using the image and the base region shown in Figure 4–a, the set of images in Figures 4–b, 4–c, and 4–d show the results of fuzzyfication by the Eu-

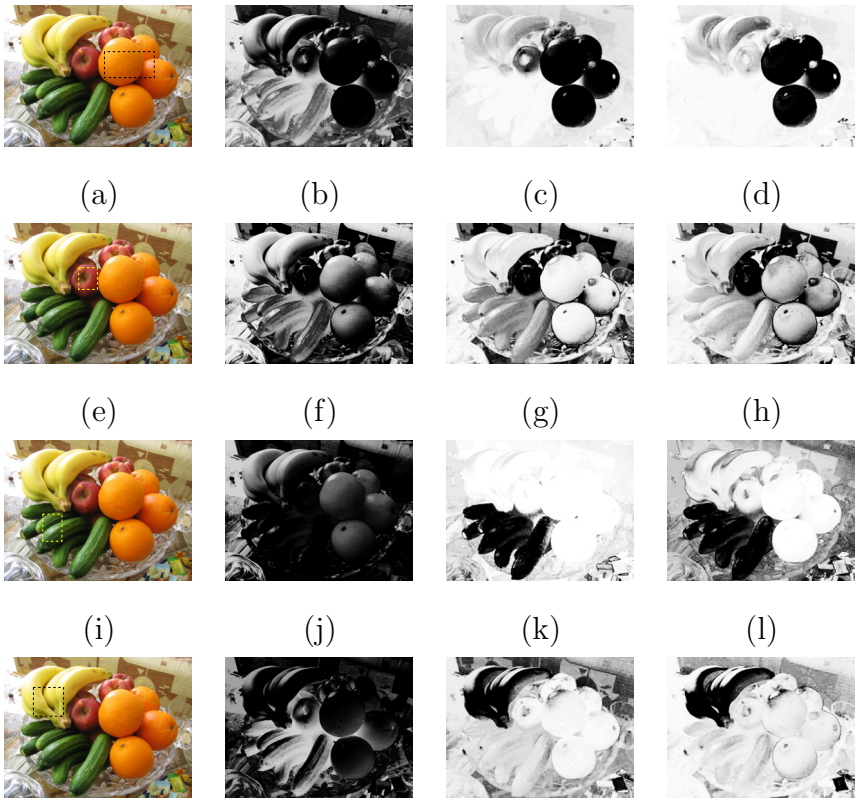


Fig. 2. Comparison of fuzzyfication methods, from left to right, original images adopted from [31] along with the base region, the Euclidean distance, the Mahalonobis distance, and the reconstruction error.

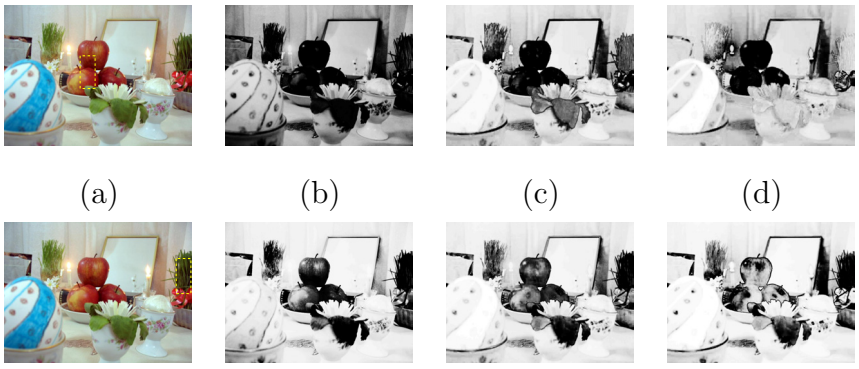


Fig. 3. Comparison of the robustness of fuzzyfication methods against non–homogeneous base regions. From left to right, original images adopted from [31] along with the base region, the Euclidean distance, the Mahalonobis distance, and the reconstruction error. (Original image shows a traditional Iranian *Seven–S* tablecloth.).

clidean distance, the Mahalonobis distance, and the reconstruction error, respectively: while p is set to the members of $\{0.9, 0.8, \cdots, 0.1\}$ iteratively, in each case. In each set of images the top left image shows the result of selecting the largest value of p (0.9), while the bottom right image belongs to the smallest choice of p (0.1). Comparing the case of $p = 0.9$ in the three sets, shows another shortcoming of the Euclidean distance–based fuzzyfication, in which increasing p results in inclusion of irrelevant points in the cluster. In the case of $p = 0.9$ for the two other methods, the fuzzyfication result contains holes of low values corresponding to the yellow flowers, which are not the members of the clusters for sure, but in the Euclidean distance the holes are filled. Comparing the sequence of fuzzyfication results in the three methods shows another promising result of the Mahalonobis distance and reconstruction error. In contrast with the Euclidean distance, in which by decreasing p, the membership function declines in all points, in the two other methods, decreasing p forces the cluster to tune more fine, rejecting less typical members and drawing the cluster edge around the set of best members.

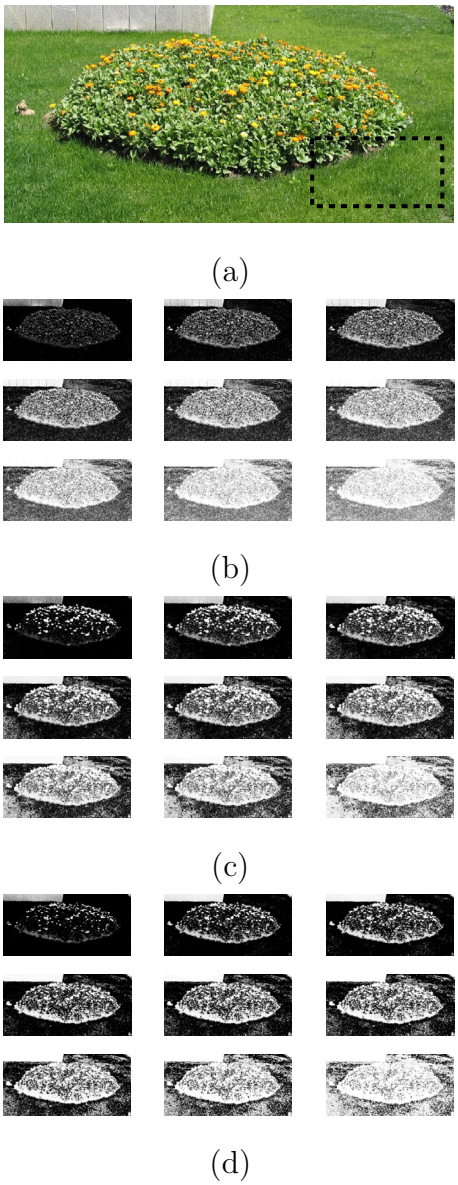
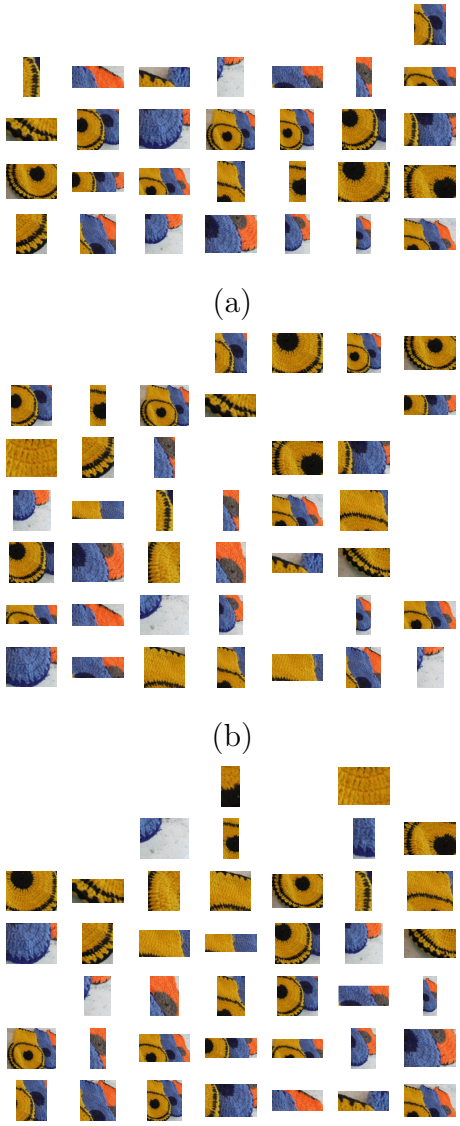


Fig. 4. Robustness of different fuzzyfication Methods against selecting different values of p. (a) Original image adopted from [31]. (b) The Euclidean distance. (c) The Mahalonobis distance. (d) The reconstruction error.

Figure 5 shows the sample image, out of which 49 random rectangular regions are selected to compare different homogeneity criteria. Figure 6 shows the randomly selected regions sorted in terms of different homogeneity criteria in a descending fashion. In each set of regions, the top left image is known to be the most homogeneous, while the bottom right region is believed to have the poorest homogeneity. In the above discussion we call the regions by the order of their homogeneity computed by each method. Investigating the order of regions computed

Fig. 5. Sample image for homogeneity criteria comparison. (Adopted from [31].).

by the Euclidean distance (Figure 5–a) makes t clear that the ordering does not comply with the human perception. For example compare the regions 40 to 43 with the 37-th and 38-th one. Although, the Euclidean distance shows that the 37-th and 38-th regions are more homogeneous, the human observers order them inversely. The same contradiction happens in the set of regions ordered by the Mahalonobis distance (Figure 5–b). For example compare the 15-th and the 14-th or the 28-th and the 4-th regions. Investigating Figure 5–c shows the similarity of the human perception and the reconstruction error homogeneity criteria. Looking at the set of regions ordered by the reconstruction error, from the first to the last, makes clear that the first regions ($1 \cdots 24$-th) are the regions containing almost just one dominant color, while the regions of the two and three colors are coming afterwards.



(a)

(b)

(c)

Fig. 6. Randomly selected regions of figure 5 sorted in terms of different homogeneity criteria in a descending fashion (a) The Euclidean distance. (b) The Mahalonobis distance. (c) the reconstruction error.

4 Conclusion

The three *vector–to–cluster* distance measures of Euclidean distance, Ma-

halonobis distance, and reconstruction error are compared, both as the likelihood measures and the homogeneity criteria. Although, the Euclidean distance (which is used commonly by the researchers) is the fastest method and needs the least memory capacity, it neither shows applicable fuzzyfication results, nor leads to a proper homogeneity criteria. Comparing the two other methods, the reconstruction error is proved to be more robust against improper base region selection, leading to a promising homogeneity criteria, while also needing less memory with low computational cost. In contrast with the common use of the Gaussian function as the membership function, the butter–worth low–pass is used and its higher performance is proved mathematically and experimentally.

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