

# An Adaptive Approach to Singular Point Detection in Fingerprint Images

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## Abstract

This paper presents a novel algorithm for detection of singular points, the core and delta points, in fingerprint images. The number and location of singular points, are used to classify fingerprint images in to five general groups; and therefore to narrow down the search space in large fingerprint databases. Using the proposed directional masks in the first step, detects the neighborhood of the singular points. In the second stage by implementing the proposed algorithm, an adaptive singular point detection method, is designed to extract the exact location of core and delta points. Usage of the proposed directional masks speeds up the process and the proposed adaptive singular point detection method increases the accuracy of the algorithm.

*Key words:* fingerprint classification, adaptive singular point detection, directional mask, core point, delta point.

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## 1 Introduction

One of the most important and efficient methods of personal identification is the fingerprint-based algorithms [1,2]. In high level fingerprint classification algorithms, extracting the number and precise location of singular points (SP), namely core and delta points, are of great importance. According to the number and location of these robust characteristics, fingerprints can be classified in to five main groups; arch, tented arch, right loop, left loop, and whorl. Using high-level classification process can efficiently reduces the search area in large fingerprint databases and therefore speeds up the subsequent matching algorithm. There are four main approaches to allocate SPs [7].

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- Methods based on mathematical model representation of fingerprint [3,4].  
Because of the complexity of fingerprint patterns, representation of an accurate model for these images is a difficult task that can not be achieved this type of models.
- Methods based on statistical approaches [8].  
In these methods although the usage of histogram has reduced the noise effect, however they cannot adapt themselves to different image characteristics.
- Methods based on Fourier transform [9].  
These methods are not efficient enough because of working in the frequency domain. But some have claimed to obtain fair results.
- Methods based on fingerprint structures. These are usually well applied approaches, which have been tested successfully on large databases [5,6,13].

Some approaches combine several types of the above mentioned methods and make a new combined system [10]. the approach proposed here contains two successive steps:

- Applying the proposed tools called directional masks to extract the neighborhood of SPs [14].
- Implementing the proposed adaptive SP detection approach to extract the precise location of SPs.

In the next section a new mathematical model for fingerprint images will be introduced. In Section 3 the proposed algorithm is discussed, in Section 4 experimental results are presented and finally the conclusion is stated.

## 2 Definitions

The proposed algorithm is applied using directional images. Before getting started with the algorithm it is essential to describe firstly the mathematical representation of directional images.

- every function  $\vec{F}$ , with its elements being two dimensional vector, is a *directional image* and can be described as follows:

$$\vec{F}(i, j) = (f_1(i, j), f_2(i, j)), \quad i, j \in \mathbb{Z}.$$

- the set of all directional images,  $v$ , make a *linear space* with the following property:

$$\alpha\vec{F} + \beta\vec{G} \triangleq (\alpha f_1 + \beta g_1, \alpha f_2 + \beta g_2)$$

- the *inner product* in the linear space is defined as:

$$\langle \vec{F}, \vec{G} \rangle \triangleq \sum_{i=-\infty}^{+\infty} \sum_{j=-\infty}^{+\infty} f_1(i, j)g_1(i, j) + f_2(i, j)g_2(i, j) \quad (1)$$

if the summation exists.

- the *absolute inner product* is defined as:

$$(\vec{F}, \vec{G}) \triangleq \sum_{i=-\infty}^{+\infty} \sum_{j=-\infty}^{+\infty} |f_1(i, j)g_1(i, j) + f_2(i, j)g_2(i, j)| \quad (2)$$

- the *correlation function* between two directional images is defined as:

$$C(m, n) \triangleq \langle \vec{F}(i, j), \vec{G}(i - m, j - n) \rangle. \quad (3)$$

- the *absolute correlation function* is defined as:

$$C(m, n) \triangleq (\vec{F}(i, j), \vec{G}(i - m, j - n)). \quad (4)$$

## 2.1 Directional Masks

In the image processing field, masks are powerful tools to extract features from images [12]. In this work we use directional masks to detect the *core* and *delta* neighborhood regions. By definition the core is defined as the topmost point on the innermost upward recurring ridge [8] and delta point is defined as the point of bifurcation (in a delta-like region) on a ridge splitting in two branches which extend to encompass the complete pattern area [8]. Figure 1 presents these singular points in a typical fingerprint image. To detect the core points the mask must extract rotational regions and to detect the delta points the mask should extract triangular regions, figure 2 shows two typical masks to detect the core and delta points, which each element being an angle in degrees. As it can be seen from this figure the core mask emphasizes on rotational regions while the delta mask emphasizes on triangular regions.



Fig. 1. Typical fingerprint image and its singular points.

|     |     |    |     |     |
|-----|-----|----|-----|-----|
| 45  | 0   | 0  | 0   | 135 |
| 90  | 45  | 0  | 135 | 90  |
| 90  | 90  | 90 | 90  | 90  |
| 90  | 135 | 0  | 45  | 90  |
| 135 | 0   | 0  | 0   | 45  |

(a)

|    |    |    |     |     |
|----|----|----|-----|-----|
| 60 | 60 | 90 | 110 | 110 |
| 50 | 50 | 90 | 120 | 120 |
| 40 | 40 | 90 | 130 | 130 |
| 30 | 30 | 90 | 140 | 140 |
| 0  | 0  | 0  | 0   | 0   |

(b)

Fig. 2. Typical directional masks: (a) core mask, (b) delta mask.

### 3 Proposed Algorithm

Figure 3 presents the overall block diagram of the proposed algorithm. The algorithm contains two successive stages to detect the SPs. In the first stage, the neighborhood of SPs are detected and in the second stage the exact location of SPs are extracted.

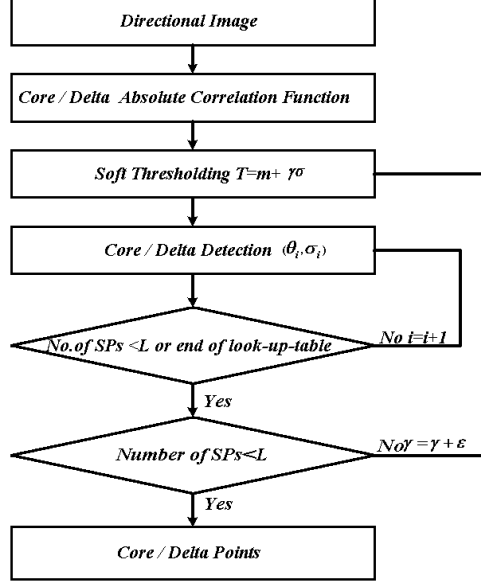


Fig. 3. Overall block diagram of the proposed algorithm.

#### 3.1 Detection of singular points neighborhood

The first stage of the proposed algorithm contains these successions:

- (1) find the value of the core/delta absolute correlatin function between the directional image and the core/delta mask according to the following formula:

$$Core(m, n) \triangleq (\vec{D}(i, j), \vec{M}_c(i - m, j - n)) \quad (5)$$

$$Delta(m, n) \triangleq (\vec{D}(i, j), \vec{M}_d(i - m, j - n)) \quad (6)$$

- (2) after finding the core/delta absolute correlation function, compute the mean,  $m$ , and variance,  $\sigma$ , of the core/delta absolute correlation function. The threshold  $T$  is computed as:

$$T(\gamma) = m + \gamma\sigma. \quad (7)$$

In this work the value of  $\gamma$  for both core and delta threshold, is initialized to 0.5.

- (3) for each pixel in the directional image, if the core/delta factor is greater than  $T(\gamma)$ , assign it as the core/delta neighborhood, else reject it.

Experimental results show that with almost all tested fingerprint images the maximum of the core/delta absolute correlation function denotes the location of core and delta points (even with presence of noise). Figure 4 presents core/delta absolute correlation function results after thresholding, for a typical fingerprint image.

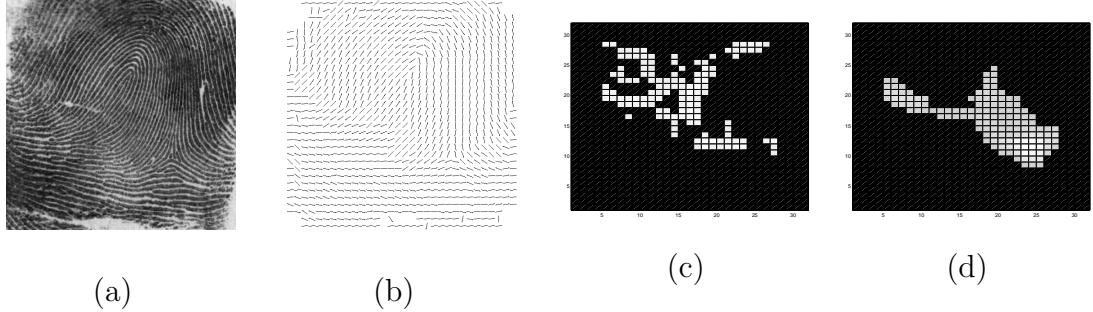


Fig. 4. Results of first stage of proposed algorithm; (a): a typical fingerprint image; (b) directional image; (c) absolute core correlation result after thresholding; (d) absolute delta correlation result after thresholding.

### 3.2 Adaptive Core/Delta Detection

For each point detected as a core neighbor (in an  $m \times m$  neighborhood ( $m=4$ ) around the point), the mean and variance of each region  $R_1, R_2, R_3, R_4$  are first computed. These regions are shown in figure 5(a). Subsequently, for each point detected as a delta neighbor, in an  $m \times m$  neighborhood ( $m=4$ ) around the point, the mean and variance for each region  $R_1, R_2, R_3$  are also computed (figure 5(b)). The point is assigned as a core if one of the following conditions is satisfied:

- $m_1 > 90, m_2 < 90, m_1 - m_2 < \theta_c(i), \sigma_1, \sigma_2 < \sigma_c(i), or$
- $m_3 < 90, m_4 > 90, m_4 - m_3 < \theta_c(i), \sigma_3, \sigma_4 < \sigma_c(i), or$
- $m_1 > 90, m_3 < 90, m_1 - m_3 < \theta_c(i), \sigma_1, \sigma_3 < \sigma_c(i), or$
- $m_2 < 90, m_4 > 90, m_4 - m_2 < \theta_c(i), \sigma_2, \sigma_4 < \sigma_c(i).$

The point is assigned as a delta point if the following conditions are both satisfied.

- $m_1 < 90, m_1 - m_3 < \theta_d(i), \sigma_1, \sigma_3 < \sigma_d(i), and$
- $m_2 > 90; 180 - (m_2 + m_3) < \theta_d(i), \sigma_2, \sigma_3 < \sigma_d(i).$

The threshold value of orientation,  $\theta(i)$ , and variance,  $\sigma(i)$ , for the core and delta points are a function of index  $i$ . We make a look-up-table (LUT) for the mean and the variance experimentally. Figure 5(c) illustrates the  $\theta - \sigma$  plots for the core and delta tables.

In each iteration if the number of core/delta points (detected by the adaptive SP detection algorithm) exceeds a threshold value of  $L$ , then another value is selected from LUT, as the threshold, otherwise the SPs are achieved. At the bottom of the library if the number of core/delta points are still greater than  $L$ , we increase  $\gamma$  (coefficient of variance) of the soft thresholding (see figure 3). The iteration continues till it converges to less than or equal to  $L$  points. Experimental results show that because of the variety of fingerprint

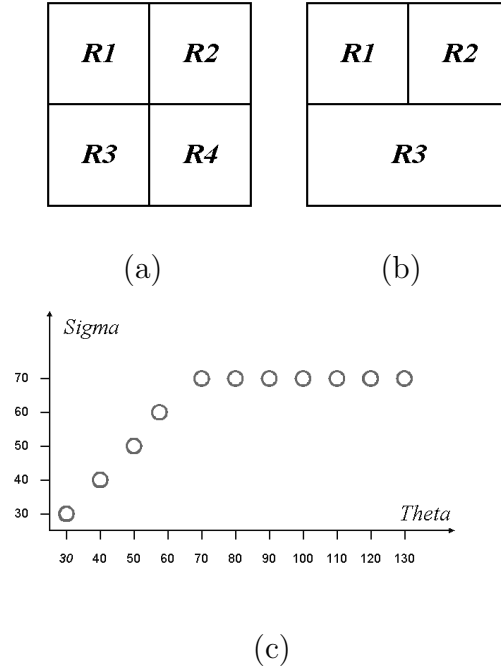


Fig. 5. (a) neighborhood region near core point; (b) neighborhood region near delta; (c) theta-sigma graph for core and delta detection;

images the selection of a fixed threshold value is not an efficient method as stated above an adaptive thresholding method is designed to extract the SPs.

Table 1

Experimental results (FP: Fingerprint Image, G: Good, VN: Very Noisy).

| <i>FP Type</i>     | <i>No. of FPs</i> | <i>Quality</i>      | <i>Core Points</i> |                         | <i>Delta Points</i> |                         |
|--------------------|-------------------|---------------------|--------------------|-------------------------|---------------------|-------------------------|
|                    |                   |                     | <i>Present</i>     | <i>Detected</i>         | <i>Present</i>      | <i>Detected</i>         |
| <i>Arch</i>        | 5                 | <i>G</i>            | <i>No</i>          | <i>Yes</i>              | <i>No</i>           | <i>Yes</i>              |
| <i>Left Loop</i>   | 10                | <i>9G &amp; 1VN</i> | <i>Yes</i>         | <i>Yes</i>              | <i>Yes</i>          | <i>9 Yes &amp; 1 No</i> |
| <i>Right Loop</i>  | 10                | <i>8G &amp; 2VN</i> | <i>Yes</i>         | <i>8 Yes &amp; 2 No</i> | <i>Yes</i>          | <i>Yes</i>              |
| <i>Tented Arch</i> | 5                 | <i>G</i>            | <i>Yes</i>         | <i>Yes</i>              | <i>Yes</i>          | <i>Yes</i>              |
| <i>Whorl</i>       | 10                | <i>9G &amp; 1VN</i> | <i>Yes</i>         | <i>8 Yes &amp; 2 No</i> | <i>Yes</i>          | <i>Yes</i>              |

## 4 Experimental Results

The approach proposed in this paper, was tested on a large fingerprint database, with different qualities, containing images with 500 dpi resolution and 256 gray-levels. To compute the directional image, the algorithm presented in [9] was used. The proposed algorithm showed very promising results even for noisy input image and show much better performances than [8]. We run the algorithm for L=5. Almost all of the SPs were detected.

In some cases the SPs were detected for arch type too, but due to the fact that arch and tented arch types exhibit different distance between SPs the images were successfully classified. Smaller values of  $\gamma$  and step better tend to better results. Figure 6 shows final results of running proposed algorithm on a typical fingerprint image. Table 1 presents the

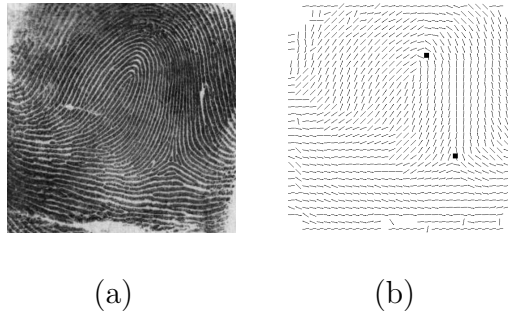


Fig. 6. Final results of running proposed algorithm on a fingerprint image: (a) query image; (b) directional image and detected singular points on it.

performance of the proposed algorithm. In our database, for some noisy images it was difficult for human to detect SPs, but in some cases the proposed algorithm detect them successfully.

## 5 Conclusion and Future Works

In this paper according to the new model of fingerprint images an adaptive approach to detect of SPs was presented. In the first step by using the proposed directional masks the SPs and their neighbor regions were detected. In the second stage by using the proposed adaptive detection algorithm the exact location of SPs were detected. The algorithm is fast and leads to better detection results compared to other available methods stated in the literature. In the future combining this method with artificial computation, results in a powerful algorithms.

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## References

- [1] Henry C. Lee and R. E. Gaensslen, *Advances in Fingerprint Technology*, Elsevier, New York, (1991).
- [2] Federal Bureau of Investigation, *The Science of Fingerprints: Classification and Uses*, U. S. Government Printing Office, Washington D.C. (1984).

- [3] B. G. Sherlock and D. M. Monro, A Model for Interpreting Fingerprint Topology, *Pattern Recognition* **26** (7) (1993) 1047–1055.
- [4] P. R. Vizcaya and L. A. Gerhardt, A Nonlinear Orientation Model for Global Description of Fingerprints, *Pattern Recognition* **29** (7) (1996) 1221–1231.
- [5] K. Karu and A. K. Jain, Fingerprint Classification, *Pattern Recognition* **29** (3) (1996) 389–404
- [6] N. K. Ratha, K. Karu, S. Chen and A. K. Jain, A Real Time Matching System for Large Fingerprint Databases, *IEEE Transactions on Pattern Analysis and Machine Intelligence* **18** (8) (August 1996) 800–813
- [7] A. K. Jain, S. Prabhakar and L. Hong, A Multichannel Approach to Fingerprint Classification, *IEEE Transactions on Pattern Analysis and Machine Intelligence* **21** (4), (April 1999) 348–358
- [8] V. S. Srinivasan and N. N. Murthy, Detection of Singular Points In Fingerprint Images, *Pattern Recognition* **25** (2) (1992) 139–153
- [9] A. P. Fitz and R. J. Green, Fingerprint Classification Using a Hexagonal Fast Fourier Transform, *Pattern Recognition* **29** (10) (1996) 1587–1597
- [10] G. T. Candela, P. J. Grother, C. I. Watson, R. A. Wilkinson, C. L. Wilson, PCASYS : A Pattern-Level Classification Automation System for Fingerprints, *U.S. DEPARTMENT of COMMERCE Technology Administration, National Institute of Standards and Technology (NIST). Advanced Systems Division, Computer Systems Laboratory, Gaithersburg, MD 20899*, (August 1995)
- [11] S. Kasaei and M. Deriche, Fingerprint Feature Enhancement using Block-Direction on Reconstructed Images, *First IEEE International Conference on Information, Communications and Signal Processing, ICICS'97, Singapore* (September 1997) 721–725
- [12] R. C. Gonzalez and P. Wintz, Digital Image Processing, *Addison-Wesley Publishing Company* (1987)
- [13] B. H. Choi et al., Core Based Fingerprint Image Classification, *ICPR'2000'S proceeding*, **2** (2000) 859–862
- [14] S. Kasaei, M. R. Rahimi, E. Pakbaznia, A Multi Stage Approach to Fingerprint Classification, *ICEE'2001'S proceeding* 14.1–14.8